MATHEMATICS II

BACHELOR IN POLITICS, LAW AND ECONOMICS

Professor: ANTONIO GARCIA ROMERO
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Academic year: 18-19
Degree course: FIRST
Semester: 2º
Category: COMPULSORY
Number of credits: 3.0
Language: English

PREREQUISITES

Concepts covered in Mathematics I, including: linear and quadratics functions; exponential and logarithmic functions; derivatives, optimization of functions of one real variable, and integration. Basics elements of algebra (pre-calculus) such as operations with powers (including negative and fractional powers) and fractions. Factorization, solving basic equations, working with inequalities and absolute values. It is also recommended to have a basic knowledge of elementary functions (polynomial, rational, power, exponential and logarithmic).

[SJM1] A good knowledge of MS-EXCEL® is highly recommendable.

SUBJECT DESCRIPTION

This subject is the second course of a one-year sequence designed to give you the intuition to think about economic ideas in mathematical terms and interpret mathematical concepts in the context of economics. Your understanding of both economics and mathematics will have improved after this sequence.

Mathematics is increasingly vital regarding the expression and communication of ideas in Economics. Good knowledge of mathematics helps to understand economic laws and their application to business management. Especially some elements of calculus and linear algebra are crucial to the study of Economics.

Economic concepts and models can often be easily and precisely described regarding mathematical notation when words and graphs would fail or mislead us. Mathematics II aims to teach you how to use mathematics to understand economics and business management. Therefore, as applications of the mathematical concepts covered in class, examples, and motivation will be drawn from important topics in both Economics and Politics. Topics covered include linear algebra, matrices, and systems of linear equations, derivatives of functions of several variables, interpretations of the derivative, convexity, constrained and unconstrained optimization. The course includes an introduction to Linear Programming by using the program Solver in EXCEL® which is crucial for modern Economic Analysis.
OBJECTIVES AND SKILLS

The main goal of this course is to provide a mathematical foundation for analysing data and drawing inferences from that analysis. Moreover, the course aims to increase the student's mastery of the deductive nature of reasoning and understanding the nature of critical thinking. To increase the student's ability in problem solving and abstract reasoning.

The objective of this course is to provide the student with part of the quantitative tools required to analyze economic, social or political problems. Regarding its contents, this first course comprises some elementary topics of Calculus of one real variable. In brief, topics covered will include basic algebra, functions of one variable and derivatives.

We classify the skills in two groups: specific and generic. Regarding the specific skills, the student will be able to:

- Define a matrix and carry out arithmetic operations involving matrices
- Solve a system of linear equations by Gaussian elimination and Gauss–Jordan elimination
- Define a determinant and evaluate $2 \times 2$ and $3 \times 3$ determinants
- Solve a system of linear equations using Cramer's rule
- Calculate the inverse of a $2 \times 2$ and $3 \times 3$ matrix
- Solve a system of linear equations by the inverse matrix method
- Solve $3 \times 3$ input/output problems.
- Use Excel to carry out elementary row operations to solve a system of linear equations and to calculate the inverse of a matrix.
- Calculate first- and second-order partial derivatives.
- Calculate differentials and incremental changes.
- Calculate marginal functions and the law of diminishing returns.
- Show that a Cobb–Douglas function is homogeneous degree $r$ and determine whether the function exhibits constant, decreasing or increasing returns to scale.
- Use partial derivatives to analyse the properties of production functions and utility functions.
- Calculate partial elasticities.
- Locate and determine the nature of stationary points for functions of several variables.
- Use Lagrange multipliers to determine maximum and minimum values for functions of two variables subject to a constraint.
- Solve linear programming problems and illustrate the results graphically.
- Use Solver in EXCEL® to find constrained maxima and minima, and to carry out what-if analysis.
- Use Excel for all of the above financial calculations.

Regarding the general skills, the student will develop the abilities:

- To address economic and political problems by means of abstract models.
- To solve the above formal models.
- To use the basic tools which are needed in the modern analysis of economic problems.

Throughout the course, the student should maintain:

- An inquisitive attitude when developing logical reasoning, being able to tell apart a proof from an example.
- An entrepreneurial and imaginative attitude towards the examples studied.
- A critical attitude towards the formal results.

METHODOLOGY

Students must work before each class. The Syllabus indicates what "Worked Examples" and "Progress Exercises" from the textbook should be read before each session. A good approach should be to read the suggested worked examples to make sure you understand the method; then try to solve the problem by yourself. The course lectures will be based on a combination of theoretical explanations and several practical exercises. Each mathematical concept will be followed immediately by one or more examples. Student participation is considered very important to acquire the skills needed to pose and solve exercises.
In this course, most of the homework will be released online through Möbius which is integrated on the IE Campus. Möbius provides an adaptive Study Plan and interactive exercises with immediate feedback. We strongly recommend that you do the exercises given as homework during the course and not leave them for a date close to the exam.

We encourage students to work in groups when solving homework problems. However, we highly recommend that each student tries, at the same time, to solve problems by himself.

<table>
<thead>
<tr>
<th>Teaching methodology</th>
<th>Weighting</th>
<th>Estimated time a student should dedicate to prepare for and participate in</th>
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<tr>
<td>Lectures</td>
<td>26.67 %</td>
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<tr>
<td>Discussions</td>
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<td>Exercises</td>
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<tr>
<td>Group work</td>
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</tr>
<tr>
<td>Other individual studying</td>
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<tr>
<td>TOTAL</td>
<td>100.0 %</td>
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PROGRAM

TOPIC 1: LINEAR ALGEBRA AND BUSINESS APPLICATIONS
FUNCTIONS OF ONE VARIABLE. [SESSIONS 1-7]
Worksheet #1
T. Bradley: Chapter 9.
EXCEL exercises: Chapter 92.

SESSION 1
Matrices. Basic operations with matrices.
Progress exercises 9.2 (page 497).

SESSION 2
Systems of Linear Equations. Elimination methods.
Readings: T. Bradley: 9.3.
Worked examples: 9.7 and 9.8.
Progress exercises 9.3 (page 503).

SESSION 3
Determinants of order 2x2. The Cramer’s rule.
Readings: T. Bradley: 9.4.1, and 9.4.2.
Progress exercises 9.4 (page 512).

SESSION 4
Determinants of order 3x3. The Cramer’s rule.
Readings: T. Bradley: 9.4.3
Worked examples: 9.14, and 9.15.
Progress exercises 9.5 (page 517).

SESSION 5
The inverse of a matrix. Elimination and cofactor methods.
Readings: T. Bradley: 9.5.
Worked examples: 9.17, and 9.18.
Progress exercises 9.6 (page 529).
EXCEL Exercises Chapter 9.3 (Read 9.6. Excel for linear Algebra).
SESSION 6
Some applications of the inverse matrix. Input/Output Analysis.
Readings: T. Bradley: 9.5.
Progress exercises 9.6 (page 529).
EXCEL Exercises Chapter 9.3 (Read 9.6. Excel for linear Algebra).

SESSION 7
Workshop #1.

TOPIC 2: FUNCTIONS OF SEVERAL VARIABLES [SESSIONS 8-18]
Worksheet #2
T. Bradley: Chapter 7.

SESSION 8
Functions of two or more variables. Partial differentiation. First order partial derivatives.
Readings: T. Bradley: 7.1.1, and 7.1.2.
Worked examples: 7.1, 7.2, 7.3.
Progress exercises: 7.1 (page 369).

SESSION 9
Second order partial derivatives. Differentials and small changes.
Readings: T. Bradley: 7.1.3 and 7.1.4.
- Worked examples: 7.4, 7.5, 7.6.a and 7.6.b.
- Progress exercises: 7.2 (page 374), and 7.3 (page 379).

SESSION 10
Business and Economic applications of Partial Differentiation (I). Production Functions and Returns to Scale.
Readings: T. Bradley: 7.2.
Worked examples: 7.7, 7.8, 7.9, 7.10, 7.11, 7.12, 7.13.
Progress exercises: 7.5 (page 393), and 7.6 (page 399).

SESSION 11
Readings: T. Bradley: 7.2.
Worked examples: 7.7, 7.8, 7.9, 7.10, 7.11, 7.12, 7.13.
Progress exercises: 7.5 (page 393), and 7.6 (page 399).
SESSION 12
Unconstrained Optimization.
Readings: T. Bradley: 7.3.1
Progress exercises 7.7 (page 403)

SESSION 13
MIDTERM EXAM
This exam will consist of TWO questions from Topics covered up to session #12. Its duration will be 30 minutes. Neither graphical nor programmable calculators are allowed for this Exam.

SESSION 14
Revenue and Profit maximization. Price discrimination.
Readings: T. Bradley: 7.3.2, and 7.3.3.
Worked examples: 7.15, 7.16, and 7.17.
Progress exercises 7.8 (page 408).

SESSION 15
Constrained optimization. The Lagrange Method. Maximizing TR subject to a budget constraint.
Readings: T. Bradley: 7.4.1, and 7.4.2.
Worked examples: 7.18
Progress exercises 7.9 (page 420).

SESSION 16
Constrained optimization. Utility functions.
Readings: T. Bradley: 7.4.3.
Worked examples: 7.18
Progress exercises 7.9 (page 420).

SESSION 17
Constrained optimization. Utility functions.
Readings: T. Bradley: 7.4.4, 7.4.5.
Worked examples: 7.19, 7.20, 7.21, 7.22, and 7.23.
Progress exercises 7.9 (page 420).

SESSION 18
Workshop #2.
TOPIC 3: LINEAR PROGRAMMING [SESSIONS 19-23]
Worksheet #3
T. Bradley: Chapter 9.
W. L. Winston: Chapter 29.
SESSION 19
Introduction to the Linear Programming Models. The graphical method.
Worked examples: 9.1, and 9.2.
Progress exercises: 9.1 (page 487).

SESSION 20
An introduction to optimization with Excel Solver.
Readings: W. L. Winston: Chapter 29.
  - EXCEL exercises (W.L. Winston) Chapter 29.

SESSION 21
Using Solver to determine the optimal product mix.
Readings: W. L. Winston: Chapter 30.
EXCEL exercises (W.L. Winston) Chapter 30.

SESSION 22
Using Solver to schedule your workforce.
Readings: W. L. Winston: Chapter 31.
EXCEL exercises (W.L. Winston) Chapter 31.

SESSION 23
Using Solver to solve transportation or distribution problems.
Readings: W. L. Winston: Chapter 32.
EXCEL exercises (W.L. Winston) Chapter 32.

SESSION 24
Final exam review.

SESSION 25
FINAL EXAM
The final exam will consist of FOUR open questions from Topics 1-3 and its duration will be 90 minutes. Neither graphical nor programmable calculators are allowed for the Final Exam.
BIBLIOGRAPHY
The followings books or articles used in this course will be:

COMPULSORY [SJM1] (BASIC LEVEL)
Title: Essential MMathematics for Economics and Business
Author: BradleyJacques, TeresaIan
Publisher / Edition / Year: John Wiley & Sons/Pearson/ 48yh / 20135
978-94-6239-035-5 / 978-94-6239-036-2 (eBook)
Medium: PRINT ELECTRONIC
Book’s student companion site: http://bcs.wiley.com/he-bcs/Books?action=index&bcsId=7971&itemId=1118358295

COMPULSORY [SJM2] (ADVANCED LEVEL)
RECOMMENDED (TOPIC 3)
Title: Microsoft Excel 2016 Data Analysis and Business Modeling
Author: WinstonJacques, Wayne L.Ian
Publisher / Edition / Year: Microsoft Press/Pearson/ 20165
978-94-6239-035-5 / 978-94-6239-036-2 (eBook)
Medium: PRINT ELECTRONIC

OTHER RESOURCES (INTERNET): Khan Academy: www.khanacademy.org
Wolfram Mathworld: http://mathworld.wolfram.com/
Integral calculator: http://integrals.wolfram.com/
Geogebra: http://www.geogebra.org
Desmos: http://www.desmos.com/calculator

EVALUATION CRITERIA

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Percentage</th>
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<tr>
<td>Class Participation</td>
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<td>Quizzes</td>
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<td>Assignment</td>
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<td>MS-EXCEL</td>
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<tr>
<td>Midterm Exam</td>
<td>20 %</td>
<td>Session 13</td>
</tr>
<tr>
<td>Final Exam</td>
<td>40 %</td>
<td>Session 25</td>
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A. CLASS PARTICIPATION
It will be worth 10% of the overall grade - students are expected to come prepared and participate actively (and voluntarily) during lectures. Moreover, the class participation will come from your results in the Problem Sets available online through Möbius. Your class grade will also be based on attendance, punctuality, engagement, and class conduct – there may be a penalty if you create a disruption, talk excessively, or use electronic devices. Your overall class participation grade will be obtained by adding the class grades across all the sessions.
B. QUIZZES.
TWO quizzes will be scheduled over the term. Please, keep in mind that If you miss a Quiz, you cannot retake it later.

C. ASSIGNMENTS.
They will be worth 1530% of the overall grade. The assignments should be done may be done both individually or in groups, as determined by the instructor, and some of them could require the use of MS-EXCEL®/MyMathLab (MML).

D. MID-TERM
It will count for 20% of the overall grade. This exam will consist of TWO open questions from Topics covered up to session #12. Moreover, neither graphical nor programmable calculators are allowed for this Exam.
Please, notice that the Mid-term date could change if the sessions are re-scheduled.

E. FINAL EXAM
It is worth 40% of the overall grade. You need to score at least 3.5 on the final exam to pass the overall course, even if you have already passed the course through the other course assessments. The final exam will consist of FOUR open questions that will cover the whole subject (i.e.: Topics 1-3). Neither graphical nor programmable calculators are allowed.

PROFESSOR BIO
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FULL-TIME PROFESSOR. AREA OF OPERATIONS AND DECISION SCIENCES. IE BUSINESS SCHOOL
Dr. Antonio García Romero is currently Assistant Professor at the Area of Operations & Decision Sciences (IE Business School). He coordinates the subjects of Mathematics at IE University. He holds a Ph.D. in Economics from the Universidad Autónoma de Madrid in 2002, a MSc. in R&D and Innovation Management (U. Carlos III), and a BSc. in Physics (U. Granada). His professional experience is a blend of academic activity, advisory positions at public administration and, recently, entrepreneurship. From 1998 to 2013, he taught Mathematics at U. Carlos III where he obtained various recognitions of teaching excellence. He also has taught at UOC (2000-03), and U. Europea de Madrid (2005-07). In addition, he participated in the International Symposium of Technology in Higher Education (U. Harvard, US). He also teaches Healthcare Innovation in Executive Education (IE Business School) and is member of the Advisory Board of the Program in Management of Healthcare Organizations. Antonio has over 20 years of experience in Studies Science, Technology and Innovation. He is member of ENID (European Network of Indicators Designers). He has published his research results in leading academic journals, and he is referee for various national and international journals. During the period 2003-13, Antonio was the Head of the Biomedical Research Policy Unit at the Regional Government of Madrid. In March 2013, he started his consultancy activity in the field of Health Innovation.

OTHER INFORMATION
Office hours: To be announced
Contact details: antonio.garcia@ie.edu BBA Faculty Room (3rd floor)
CODE OF CONDUCT IN CLASS

1. **Be on time**: Students arriving more than 5 minutes late will be marked as “Absent”. Only students that notify in advance in writing that they will be late for a specific session may be granted an exception (at the discretion of the professor).

2. **If applicable, bring your name card and strictly follow the seating chart.** It helps faculty members and fellow students learn your names.

3. **Do not leave the room during the lecture**: Students are not allowed to leave the room during lectures. If a student leaves the room during lectures, he/she will not be allowed to re-enter and, therefore, will be marked as “Absent”. Only students that notify that they have a special reason to leave the session early will be granted an exception (at the discretion of the professor).

4. **Do not engage in side conversation.** As a sign of respect toward the person presenting the lecture (the teacher as well as fellow students), side conversations are not allowed. If you have a question, raise your hand and ask it. If you do not want to ask it during the lecture, feel free to approach your teacher after class. If a student is disrupting the flow of the lecture, he/she will be asked to leave the classroom and, consequently, will be marked as “Absent”.

5. **Use your laptop for course-related purposes only.** The use of laptops during lectures must be authorized by the professor. The use of Social Media or accessing any type of content not related to the lecture is penalized. The student will be asked to leave the room and, consequently, will be marked as “Absent”.

6. **No cellular phones**: IE University implements a “Phone-free Classroom” policy and, therefore, the use of phones, tablets, etc. is forbidden inside the classroom. Failing to abide by this rule entails expulsion from the room and will be counted as one absence.

7. **Escalation policy: 1/3/5.** Items 4, 5, and 6 above entail expulsion from the classroom and the consequent marking of the student as “Absent.” IE University implements an “escalation policy”: The first time a student is asked to leave the room for disciplinary reasons (as per items 4, 5, and 6 above), the student will incur one absence, the second time it will count as three absences, and from the third time onward, any expulsion from the classroom due to disciplinary issues will entail 5 absences.